Progress of the AMIDAS Package for Reconstructing WIMP Properties

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4th International Workshop on
Dark Matter, Dark Energy, and Matter-Antimatter Asymmetry

December 31, 2016

AMIDAS package
– **A** **M**odel-**I**ndependent **D**ata **A**nalysis **S**ystem

AMIDAS package

- AMIDAS: A Model-Independent Data Analysis System
  for direct Dark Matter detection experiments and phenomenology

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  - DAMNED Dark Matter Web Tool (ILIAS Project)
    `http://pisrv0.pit.physik.uni-tuebingen.de/darkmatter/amidas/`
    [CLS, AIP Conf. Proc. 1200, 1031; arXiv:0910.1971; Phys. Dark Univ. 5-6, 240 (2014)]

  - TiResearch (Taiwan interactive Research)
    `http://www.tir.tw/phys/hep/dm/amidas/`

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  - Online interactive simulation/data analysis system
  - Full Monte Carlo simulations
  - Theoretical estimations
  - Real/pseudo- data analyses

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Chung-Lin Shan

Dark Matter Online Tools   DAMNED   AMIDAS: restart, new window, last results

Established: January 11, 2009
Last upgraded: March 18, 2015

List of AMIDAS functions

Choose one of the following functions

- ○ Generation of WIMP signals with/without background events
- ⦿ (Bayesian) reconstruction of the one-dimensional velocity distribution function of halo WIMPs new
- ○ Determination of the WIMP mass
- ○ Estimation of the spin-independent (SI) WIMP-nucleon coupling
- ○ Determinations of ratios between different WIMP-nucleon couplings/cross sections

Motivation

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- Differential event rate for elastic WIMP-nucleus scattering

$$\frac{dR}{dQ} = \mathcal{A} F^2(Q) \int_{v_{\rm min}(Q)}^{v_{\rm max}} \left[\frac{f_1(v)}{v} \right] dv$$

Here

$$v_{\rm min}(Q) = \alpha \sqrt{Q}$$

is the minimal incoming velocity of incident WIMPs that can deposit the recoil energy Q in the detector,

$$\mathcal{A} \equiv \frac{\rho_0 \sigma_0}{2 m_\chi m_{\rm r,N}^2} \qquad \alpha \equiv \sqrt{\frac{m_{\rm N}}{2 m_{\rm r,N}^2}} \qquad m_{\rm r,N} = \frac{m_\chi m_{\rm N}}{m_\chi + m_{\rm N}}$$

ρ_0: WIMP density near the Earth

σ_0: total cross section ignoring the form factor suppression

$F(Q)$: elastic nuclear form factor

$f_1(v)$: one-dimensional velocity distribution of halo WIMPs

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Particle physics

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Astrophysics

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Reconstruction of the 1-D WIMP velocity distribution

With measured recoil energies

Reconstruction of the 1-D WIMP velocity distribution

❏ Normalized one-dimensional WIMP velocity distribution function

$$f_1(v) = \mathcal{N} \left\{ -2Q \cdot \frac{d}{dQ} \left[\frac{1}{F^2(Q)} \left(\frac{dR}{dQ} \right) \right] \right\}_{Q=v^2/\alpha^2}$$

$$\mathcal{N} = \frac{2}{\alpha} \left\{ \int_0^\infty \frac{1}{\sqrt{Q}} \left[\frac{1}{F^2(Q)} \left(\frac{dR}{dQ} \right) \right] dQ \right\}^{-1}$$

❏ Moments of the velocity distribution function

$$\langle v^n \rangle = \mathcal{N}(Q_{\rm thre}) \left(\frac{\alpha^{n+1}}{2} \right) \left[\frac{2Q_{\rm thre}^{(n+1)/2}}{F^2(Q_{\rm thre})} \left(\frac{dR}{dQ} \right)_{Q=Q_{\rm thre}} + (n+1) I_n(Q_{\rm thre}) \right]$$

$$\mathcal{N}(Q_{\rm thre}) = \frac{2}{\alpha} \left[\frac{2Q_{\rm thre}^{1/2}}{F^2(Q_{\rm thre})} \left(\frac{dR}{dQ} \right)_{Q=Q_{\rm thre}} + I_0(Q_{\rm thre}) \right]^{-1}$$

$$I_n(Q_{\rm thre}) = \int_{Q_{\rm thre}}^\infty Q^{(n-1)/2} \left[\frac{1}{F^2(Q)} \left(\frac{dR}{dQ} \right) \right] dQ$$

[M. Drees and CLS, JCAP 0706, 011 (2007)]

Reconstruction of the 1-D WIMP velocity distribution

- Ansatz: the measured recoil spectrum in the nth Q-bin

$$\left(\frac{dR}{dQ}\right)_{\text{expt},\ Q \simeq Q_n} \equiv r_n\, e^{k_n (Q - Q_{s,n})} \qquad r_n \equiv \frac{N_n}{b_n}$$

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- Logarithmic slope and shifted point in the nth Q-bin

$$\overline{Q-Q_n}|_n \equiv \frac{1}{N_n}\sum_{i=1}^{N_n}(Q_{n,i}-Q_n) = \left(\frac{b_n}{2}\right)\coth\left(\frac{k_n b_n}{2}\right) - \frac{1}{k_n}$$

$$Q_{s,n} = Q_n + \frac{1}{k_n}\ln\left[\frac{\sinh(k_n b_n/2)}{k_n b_n/2}\right]$$

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$$Q_{s,n} = Q_n + \frac{1}{k_n} \ln\left[\frac{\sinh(k_n b_n/2)}{k_n b_n/2}\right]$$

- Reconstructing the one-dimensional WIMP velocity distribution

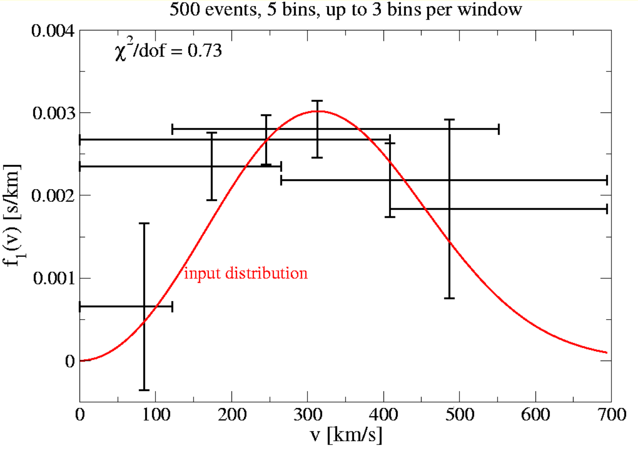
$$f_1(v_{s,n}) = \mathcal{N}\left[\frac{2 Q_{s,n} r_n}{F^2(Q_{s,n})}\right]\left[\frac{d}{dQ} \ln F^2(Q)\bigg|_{Q = Q_{s,n}} - k_n\right]$$

$$\mathcal{N} = \frac{2}{\alpha}\left[\sum_a \frac{1}{\sqrt{Q_a}\, F^2(Q_a)}\right]^{-1} \qquad v_{s,n} = \alpha\sqrt{Q_{s,n}}$$

[M. Drees and CLS, JCAP 0706, 011 (2007)]

Reconstruction of the 1-D WIMP velocity distribution

- Reconstructed $f_{1,\mathrm{rec}}(v_{s,n})$
  (^{76}Ge, 500 events, 5 bins, up to 3 bins per window)

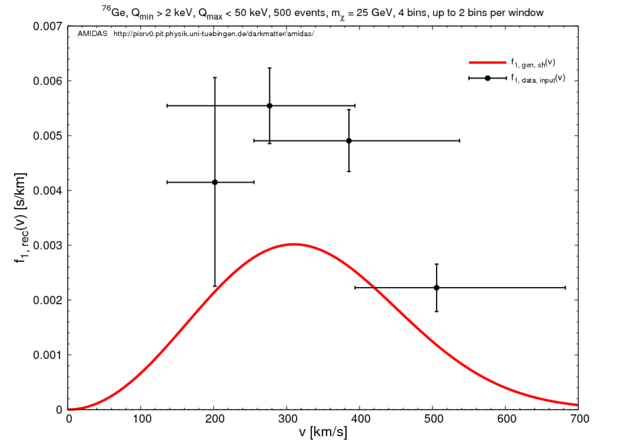


[M. Drees and CLS, JCAP 0706, 011 (2007)]

With a non-negligible threshold energy

Reconstruction of $f_1(v)$ with a non-negligible threshold energy

- Reconstructed $f_{1,\rm rec}(v_{s,n})$ with a non-negligible threshold energy (^{76}Ge, 2 - 50 keV, 500 events, $m_\chi = 25$ GeV)



[CLS, IJMPD 24, 1550090 (2015)]

Reconstruction of $f_1(v)$ with a non-negligible threshold energy

- Modification of the renormalization constant

$$\mathcal{N} = \frac{2}{\alpha}\left[\tilde{f}_{1,\rm rec}(v^*_{\rm min})\, Q_{\rm min}^{1/2} + \frac{2Q_{\rm min}^{1/2}}{F^2(Q_{\rm min})}\left(\frac{dR}{dQ}\right)_{{\rm expt},\ Q=Q_{\rm min}} + I_0(Q_{\rm min}, Q^*_{\rm max})\right]^{-1}$$

where

$$\tilde{f}_{1,\rm rec}(v^*_{\rm min}) \equiv \left[\frac{2Q_{\rm min}\, r(Q_{\rm min})}{F^2(Q_{\rm min})}\right]\left[\frac{d}{dQ}\ln F^2(Q)\bigg|_{Q=Q_{\rm min}} - k_1\right]$$

$$\left(\frac{dR}{dQ}\right)_{{\rm expt},\ Q=Q_{\rm min}} = r_1\, e^{k_1(Q_{\rm min}-Q_{s,1})} \equiv r(Q_{\rm min})$$

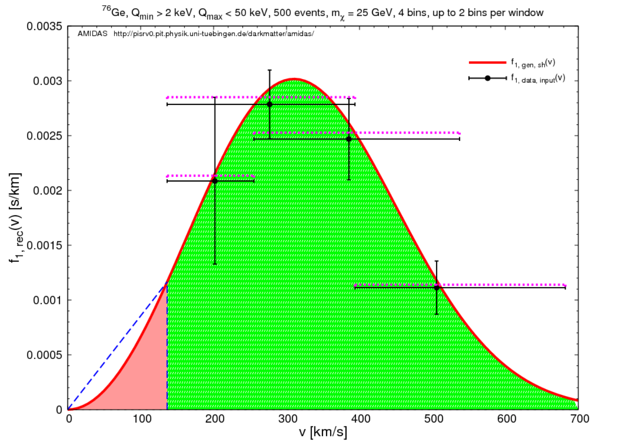
$$I_n(Q_{\rm min}, Q^*_{\rm max}) = \int_{Q_{\rm min}}^{Q^*_{\rm max}} Q^{(n-1)/2}\left[\frac{1}{F^2(Q)}\left(\frac{dR}{dQ}\right)\right] dQ \to \sum_a \frac{Q_a^{(n-1)/2}}{F^2(Q_a)}$$

$$Q^*_{\rm max} \equiv \min\left(Q_{\rm max},\ Q_{\rm max,kin} = \frac{v^2_{\rm max}}{\alpha^2}\right)$$

[CLS, IJMPD 24, 1550090 (2015)]

Reconstruction of $f_1(v)$ with a non-negligible threshold energy

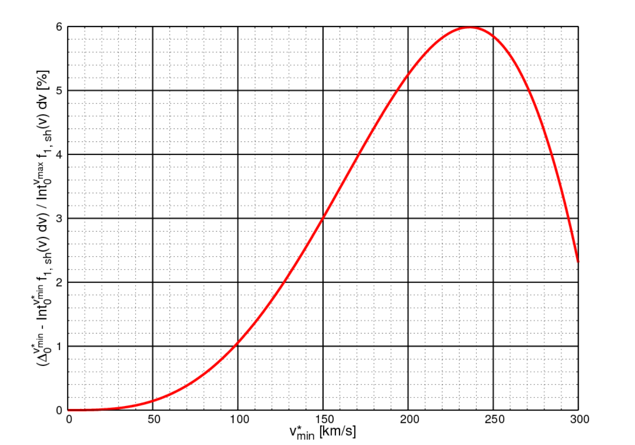
- Reconstructed $f_{1,\text{rec}}(v_{s,n})$ with the input WIMP mass
  (^{76}Ge, 2 - 50 keV, 500 events, $m_\chi = 25$ GeV)



[CLS, IJMPD 24, 1550090 (2015)]

Reconstruction of $f_1(v)$ with a non-negligible threshold energy

- Theoretical bias estimate of $\left[\Delta_0^{v^*_{\rm min}} - \int_0^{v^*_{\rm min}} f_1(v)\, dv\right] / \int_0^{v_{\rm max}} f_1(v)\, dv$



[CLS, IJMPD 24, 1550090 (2015)]

With a model of the WIMP velocity distribution – Bayesian analysis

Bayesian reconstruction of $f_1(v)$

❏ Bayesian analysis

$$p(\Theta|\text{data}) = \frac{p(\text{data}|\Theta)}{p(\text{data})} \cdot p(\Theta)$$

➢ Θ: $\left\{a_1, a_2, \cdots, a_{N_{\rm Bayesian}}\right\}$, a specified (combination of the) value(s) of the fitting parameter(s)

➢ $p(\Theta)$: prior probability, our degree of belief about Θ being the true value(s) of fitting parameter(s), often given in form of the (multiplication of the) probability distribution(s) of the fitting parameter(s)

➢ $p(\text{data}|\Theta)$: the probability of the observed result, once the specified (combination of the) value(s) of the fitting parameter(s) happens, usually be described by the "likelihood" function of Θ, $\mathcal{L}(\Theta)$.

➢ $p(\text{data})$: evidence, the total probability of obtaining the particular set of data

➢ $p(\Theta|\text{data})$: posterior probability density function for Θ, the probability of that the specified (combination of the) value(s) of the fitting parameter(s) happens, given the observed result

Bayesian reconstruction of $f_1(v)$

- Probability distribution functions for $\mathrm{p}(\Theta)$
  - Without prior knowledge about the fitting parameter
    - Flat-distributed

$$\mathrm{p}_i(a_i) = 1 \qquad \text{for } a_{i,\min} \le a_i \le a_{i,\max}$$

  - With prior knowledge about the fitting parameter
    - Around a theoretical predicted/estimated or experimental measured value $\mu_{a,i}$
    - With (statistical) uncertainties $\sigma_{a,i}$
    - Gaussian-distributed

$$\mathrm{p}_i(a_i; \mu_{a,i}, \sigma_{a,i}) = \frac{1}{\sqrt{2\pi}\,\sigma_{a,i}}\, e^{-(a_i - \mu_{a,i})^2/2\sigma_{a,i}^2}$$

[CLS, JCAP 1408, 009 (2014)]

Bayesian reconstruction of $f_1(v)$

- Likelihood function for p(data|Θ)
  - Theoretical one-dimensional WIMP velocity distribution function: $f_{1,\rm th}(v; a_1, a_2, \cdots, a_{N_{\rm Bayesian}})$
  - Assuming that the reconstructed data points are Gaussian-distributed around the theoretical predictions

$$\mathcal{L}\Big(f_{1,\rm rec}(v_{s,\mu}),\ \mu = 1,\ 2,\ \cdots,\ W;\ a_i,\ i = 1,\ 2,\ \cdots,\ N_{\rm Bayesian}\Big)$$

$$\equiv \prod_{\mu=1}^{W} {\rm Gau}\Big(v_{s,\mu}, f_{1,\rm rec}(v_{s,\mu}), \sigma_{f_1,s,\mu}; a_1, a_2, \cdots, a_{N_{\rm Bayesian}}\Big)$$

with

$${\rm Gau}\Big(v_{s,\mu}, f_{1,\rm rec}(v_{s,\mu}), \sigma_{f_1,s,\mu}; a_1, a_2, \cdots, a_{N_{\rm Bayesian}}\Big)$$

$$\equiv \frac{1}{\sqrt{2\pi}\,\sigma_{f_1,s,\mu}}\, e^{-\left[f_{1,\rm rec}(v_{s,\mu}) - f_{1,\rm th}(v_{s,\mu}; a_1, a_2, \cdots, a_{N_{\rm Bayesian}})\right]^2 / 2\sigma_{f_1,s,\mu}^2}$$

[CLS, JCAP 1408, 009 (2014)]

Bayesian reconstruction of $f_1(v)$

- Input and fitting one-dimensional WIMP velocity distribution functions
  - "One-parameter" shifted Maxwellian velocity distribution

$$f_{1,\mathrm{sh},v_0}(v) = \frac{1}{\sqrt{\pi}} \left(\frac{v}{v_0 v_\mathrm{e}} \right) \left[e^{-(v-v_\mathrm{e})^2/v_0^2} - e^{-(v+v_\mathrm{e})^2/v_0^2} \right] \qquad v_\mathrm{e} = 1.05\, v_0$$

  - Shifted Maxwellian velocity distribution

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  - "Variated" shifted Maxwellian velocity distribution

$$f_{1,\mathrm{sh},\Delta v}(v) = \frac{1}{\sqrt{\pi}} \left[\frac{v}{v_0 \, (v_0 + \Delta v)} \right] \left\{ e^{-[v-(v_0+\Delta v)]^2/v_0^2} - e^{-[v+(v_0+\Delta v)]^2/v_0^2} \right\}$$

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  - Simple Maxwellian velocity distribution

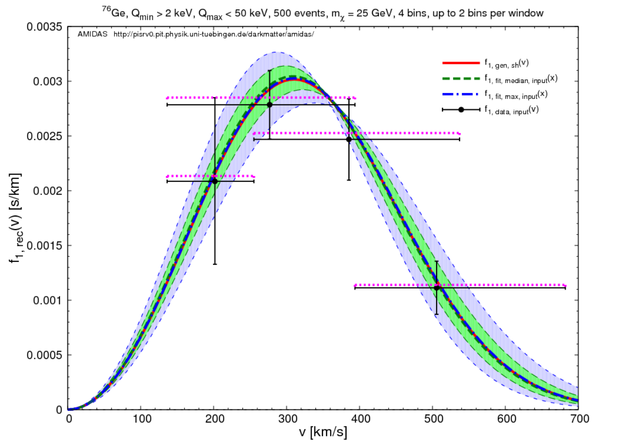
$$f_{1,\mathrm{Gau}}(v) = \frac{4}{\sqrt{\pi}} \left(\frac{v^2}{v_0^3}\right) e^{-v^2/v_0^2}$$

  - "Modified" simple Maxwellian velocity distribution

$$f_{1,\mathrm{Gau},k}(v) = \frac{v^2}{N_{f,k}} \left(e^{-v^2/kv_0^2} - e^{-v_\mathrm{max}^2/kv_0^2}\right)^k \qquad \text{for } v \le v_\mathrm{max}$$

Bayesian reconstruction of $f_1(v)$

- Reconstructed $f_{1,\text{Bayesian}}(v)$ with the input WIMP mass
  (^{76}Ge, 2 - 50 keV, 500 events, $m_\chi = 25$ GeV, $f_{1,\text{sh},v_0}(v) \Rightarrow f_{1,\text{sh},v_0}(v)$, flat-dist.)



[CLS, IJMPD 24, 1550090 (2015)]

Determination of the WIMP mass

Determination of the WIMP mass

- Estimating the moments of the WIMP velocity distribution

$$\langle v^n \rangle = \alpha^n \left[\frac{2Q_{\rm min}^{1/2} r_{\rm min}}{F^2(Q_{\rm min})} + I_0 \right]^{-1} \left[\frac{2Q_{\rm min}^{(n+1)/2} r_{\rm min}}{F^2(Q_{\rm min})} + (n+1) I_n \right]$$

$$I_n = \sum_a \frac{Q_a^{(n-1)/2}}{F^2(Q_a)} \qquad r_{\rm min} = \left(\frac{dR}{dQ} \right)_{{\rm expt},\, Q=Q_{\rm min}} = r_1 \, e^{k_1 (Q_{\rm min} - Q_{s,1})}$$

[M. Drees and CLS, JCAP 0706, 011 (2007)]

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[M. Drees and CLS, JCAP 0706, 011 (2007)]

- Determining the WIMP mass

$$m_\chi \big|_{\langle v^n \rangle} = \frac{\sqrt{m_X m_Y} - m_X \mathcal{R}_n}{\mathcal{R}_n - \sqrt{m_X / m_Y}}$$

$$\mathcal{R}_n = \left[\frac{2Q_{{\rm min},X}^{(n+1)/2} r_{{\rm min},X} / F_X^2(Q_{{\rm min},X}) + (n+1) I_{n,X}}{2Q_{{\rm min},X}^{1/2} r_{{\rm min},X} / F_X^2(Q_{{\rm min},X}) + I_{0,X}} \right]^{1/n} (X \longrightarrow Y)^{-1} \qquad (n \neq 0)$$

[CLS and M. Drees, arXiv:0710.4296]

Determination of the WIMP mass

- Estimating the moments of the WIMP velocity distribution

$$\langle v^n \rangle = \alpha^n \left[\frac{2Q_{\rm min}^{1/2} r_{\rm min}}{F^2(Q_{\rm min})} + I_0 \right]^{-1} \left[\frac{2Q_{\rm min}^{(n+1)/2} r_{\rm min}}{F^2(Q_{\rm min})} + (n+1) I_n \right]$$

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[M. Drees and CLS, JCAP 0706, 011 (2007)]

- Determining the WIMP mass

$$m_\chi \big|_{\langle v^n \rangle} = \frac{\sqrt{m_X m_Y} - m_X \mathcal{R}_n}{\mathcal{R}_n - \sqrt{m_X / m_Y}}$$

$$\mathcal{R}_n = \left[\frac{2Q_{{\rm min},X}^{(n+1)/2} r_{{\rm min},X} / F_X^2(Q_{{\rm min},X}) + (n+1) I_{n,X}}{2Q_{{\rm min},X}^{1/2} r_{{\rm min},X} / F_X^2(Q_{{\rm min},X}) + I_{0,X}} \right]^{1/n} (X \longrightarrow Y)^{-1} \qquad (n \neq 0)$$

[CLS and M. Drees, arXiv:0710.4296]

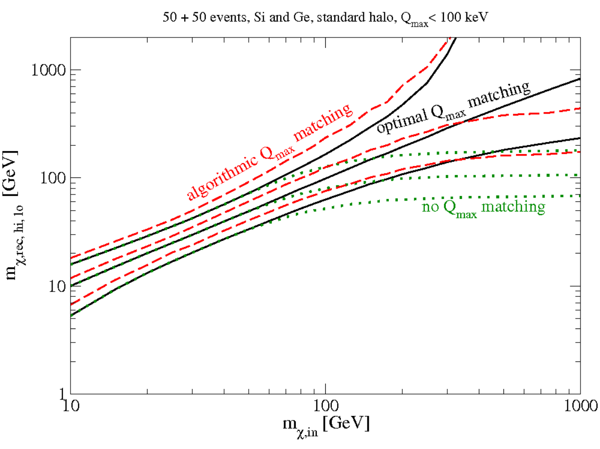
- Assuming a dominant SI scalar WIMP-nucleus interaction

$$m_\chi \big|_\sigma = \frac{(m_X / m_Y)^{5/2} \, m_Y - m_X \mathcal{R}_\sigma}{\mathcal{R}_\sigma - (m_X / m_Y)^{5/2}} \qquad \mathcal{R}_\sigma = \frac{\mathcal{E}_Y}{\mathcal{E}_X} \left[\frac{2Q_{{\rm min},X}^{1/2} r_{{\rm min},X} / F_X^2(Q_{{\rm min},X}) + I_{0,X}}{2Q_{{\rm min},Y}^{1/2} r_{{\rm min},X} / F_Y^2(Q_{{\rm min},Y}) + I_{0,Y}} \right]$$

[M. Drees and CLS, JCAP 0806, 012 (2008)]

Determination of the WIMP mass

- Reconstructed $m_{\chi,\mathrm{rec}}$
  ($^{28}\mathrm{Si} + {}^{76}\mathrm{Ge}$, $Q_{\mathrm{max}} < 100$ keV, 2×50 events)



[M. Drees and CLS, JCAP 0806, 012 (2008)]

Determinations of the WIMP-nucleon couplings

Estimation of the SI scalar WIMP-nucleon coupling

Estimation of the SI scalar WIMP-nucleon coupling

- Spin-independent (SI) scalar WIMP-nucleus cross section

$$\sigma_0^{\rm SI} = \left(\frac{4}{\pi}\right) m_{\rm r,N}^2 \left[Z f_{\rm p} + (A - Z) f_{\rm n}\right]^2 \simeq \left(\frac{4}{\pi}\right) m_{\rm r,N}^2 A^2 |f_{\rm p}|^2 = A^2 \left(\frac{m_{\rm r,N}}{m_{\rm r,p}}\right)^2 \sigma_{\chi \rm p}^{\rm SI}$$

$$\sigma_{\chi \rm p}^{\rm SI} = \left(\frac{4}{\pi}\right) m_{\rm r,p}^2 |f_{\rm p}|^2$$

$f_{\rm (p,n)}$: effective SI scalar WIMP-proton/neutron couplings

Estimation of the SI scalar WIMP-nucleon coupling

- Spin-independent (SI) scalar WIMP-nucleus cross section

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$$\left(\frac{dR}{dQ}\right)_{{\rm expt},\, Q=Q_{\rm min}} = \frac{\mathcal{E} \rho_0 A^2}{2 m_\chi m_{\rm r,p}^2} \left[\left(\frac{4}{\pi}\right) m_{\rm r,p}^2 |f_{\rm p}|^2\right] F^2(Q_{\rm min}) \left\{ m_{\rm r,N} \sqrt{\frac{2}{m_{\rm N}}} \left[\frac{2 Q_{\rm min}^{1/2} r_{\rm min}}{F^2(Q_{\rm min})} + I_0\right]^{-1} \left[\frac{2\, r_{\rm min}}{F^2(Q_{\rm min})}\right] \right\}$$

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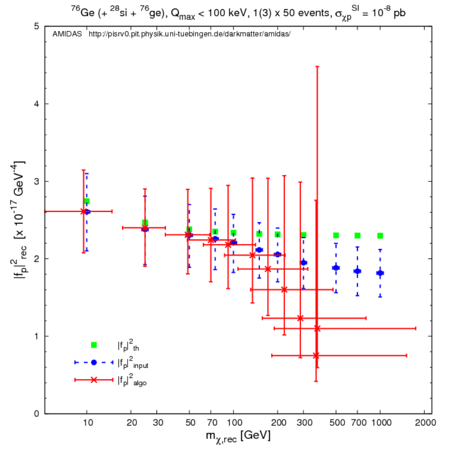
- Estimating the SI scalar WIMP-nucleon coupling

$$|f_{\rm p}|^2 = \frac{1}{\rho_0} \left[\frac{\pi}{4\sqrt{2}} \left(\frac{1}{\mathcal{E}_Z A_Z^2 \sqrt{m_Z}}\right)\right] \left[\frac{2 Q_{{\rm min},Z}^{1/2} r_{{\rm min},Z}}{F_Z^2(Q_{{\rm min},Z})} + I_{0,Z}\right] (m_\chi + m_Z)$$

[M. Drees and CLS, PoS IDM2008, 110 (2008)]

Estimation of the SI scalar WIMP-nucleon coupling

- Reconstructed $|f_\mathrm{p}|^2_\mathrm{rec}$ vs. reconstructed $m_{\chi,\mathrm{rec}}$
  ($^{76}\mathrm{Ge}\,(+\ ^{28}\mathrm{Si} +\ ^{76}\mathrm{Ge})$, $Q_\mathrm{max} < 100$ keV, $\sigma_{\chi\mathrm{p}}^\mathrm{SI} = 10^{-8}$ pb, $1(3) \times 50$ events)



[CLS, arXiv:1103.0481]

Determination of the ratio of SD WIMP-nucleon couplings

- Spin-dependent (SD) axial-vector WIMP-nucleus cross section

$$\sigma_0^{\rm SD} = \left(\frac{32}{\pi}\right) G_F^2\, m_{\rm r,N}^2 \left(\frac{J+1}{J}\right) \left[\langle S_{\rm p}\rangle a_{\rm p} + \langle S_{\rm n}\rangle a_{\rm n}\right]^2$$

$$\sigma_{\chi{\rm p/n}}^{\rm SD} = \left(\frac{32}{\pi}\right) G_F^2\, m_{\rm r,p/n}^2 \cdot \left(\frac{3}{4}\right) a_{\rm p/n}^2$$

J: total nuclear spin

$\langle S_{\rm (p,n)}\rangle$: expectation values of the proton/neutron group spin

$a_{\rm (p,n)}$: effective SD axial-vector WIMP-proton/neutron couplings

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$a_{\rm (p,n)}$: effective SD axial-vector WIMP-proton/neutron couplings

- Determining the ratio of two SD axial-vector WIMP-nucleon couplings

$$\left(\frac{a_{\rm n}}{a_{\rm p}}\right)_{\pm,n}^{\rm SD} = -\,\frac{\langle S_{\rm p}\rangle_X \pm \langle S_{\rm p}\rangle_Y \mathcal{R}_{J,n}}{\langle S_{\rm n}\rangle_X \pm \langle S_{\rm n}\rangle_Y \mathcal{R}_{J,n}}$$

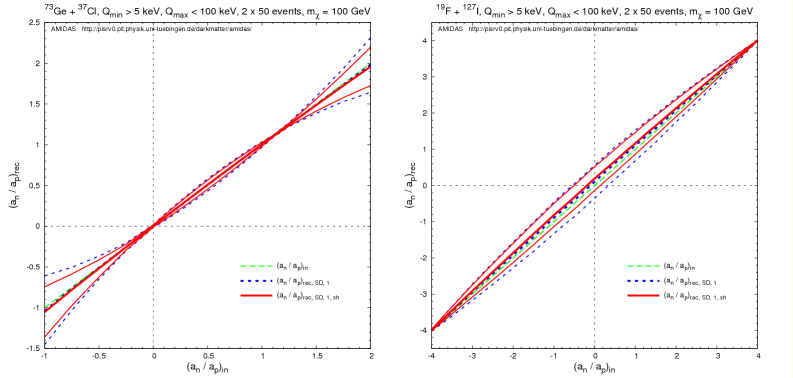
$$\mathcal{R}_{J,n} \equiv \left[\left(\frac{J_X}{J_X+1}\right)\left(\frac{J_Y+1}{J_Y}\right)\frac{\mathcal{R}_\sigma}{\mathcal{R}_n}\right]^{1/2} \qquad (n \neq 0)$$

[M. Drees and CLS, arXiv:0903.3300]

Determination of the ratio of SD WIMP-nucleon couplings

- Reconstructed $(a_n/a_p)^{SD}_{rec,1}$

  (^{73}Ge + ^{37}Cl and ^{19}F + ^{127}I, $Q_{min} > 5$ keV, $Q_{max} < 100$ keV, 2×50 events, $m_\chi = 100$ GeV)



[CLS, JCAP 1107, 005 (2011)]

Determinations of ratios of WIMP-nucleon cross sections

- Differential rate for combined SI and SD cross sections

$$\left(\frac{dR}{dQ}\right)_{\text{expt},\,Q=Q_{\text{min}}} = \mathcal{E}\left(\frac{\rho_0 \sigma_0^{\text{SI}}}{2 m_\chi m_{\text{r,N}}^2}\right)\left[F_{\text{SI}}^2(Q) + \left(\frac{\sigma_{\chi\text{p}}^{\text{SD}}}{\sigma_{\chi\text{p}}^{\text{SI}}}\right)\mathcal{C}_{\text{p}} F_{\text{SD}}^2(Q)\right]\int_{v_{\text{min}}}^{v_{\text{max}}}\left[\frac{f_1(v)}{v}\right]dv$$

$$\mathcal{C}_{\text{p}} \equiv \frac{4}{3}\left(\frac{J+1}{J}\right)\left[\frac{\langle S_{\text{p}}\rangle + (a_{\text{n}}/a_{\text{p}})\langle S_{\text{n}}\rangle}{A}\right]^2$$

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- Determining the ratio of two WIMP-proton cross sections

$$\frac{\sigma_{\chi\text{p}}^{\text{SD}}}{\sigma_{\chi\text{p}}^{\text{SI}}} = \frac{F_{\text{SI},Y}^2(Q_{\text{min},Y})\mathcal{R}_{m,XY} - F_{\text{SI},X}^2(Q_{\text{min},X})}{\mathcal{C}_{\text{p},X} F_{\text{SD},X}^2(Q_{\text{min},X}) - \mathcal{C}_{\text{p},Y} F_{\text{SD},Y}^2(Q_{\text{min},Y})\mathcal{R}_{m,XY}}$$

$$\mathcal{R}_{m,XY} \equiv \left(\frac{r_{\text{min},X}}{\mathcal{E}_X}\right)\left(\frac{\mathcal{E}_Y}{r_{\text{min},Y}}\right)\left(\frac{m_Y}{m_X}\right)^2$$

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$$\mathcal{R}_{m,XY} \equiv \left(\frac{r_{\text{min},X}}{\mathcal{E}_X}\right)\left(\frac{\mathcal{E}_Y}{r_{\text{min},Y}}\right)\left(\frac{m_Y}{m_X}\right)^2$$

- Determining the ratio of two SD axial-vector WIMP-nucleon couplings

$$\left(\frac{a_\text{n}}{a_\text{p}}\right)_\pm^{\text{SI+SD}} = \frac{-\left(c_{\text{p},X}s_{\text{n/p},X} - c_{\text{p},Y}s_{\text{n/p},Y}\right) \pm \sqrt{c_{\text{p},X}c_{\text{p},Y}}\left|s_{\text{n/p},X} - s_{\text{n/p},Y}\right|}{c_{\text{p},X}s_{\text{n/p},X}^2 - c_{\text{p},Y}s_{\text{n/p},Y}^2}$$

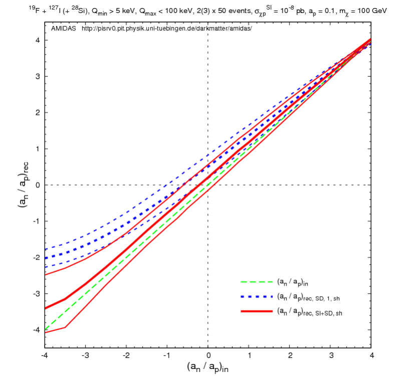
$$c_{\text{p},X} \equiv \frac{4}{3}\left(\frac{J_X+1}{J_X}\right)\left[\frac{\langle S_\text{p}\rangle_X}{A_X}\right]^2\left[F_{\text{SI},Z}^2(Q_{\text{min},Z})\mathcal{R}_{m,YZ} - F_{\text{SI},Y}^2(Q_{\text{min},Y})\right]F_{\text{SD},X}^2(Q_{\text{min},X})$$

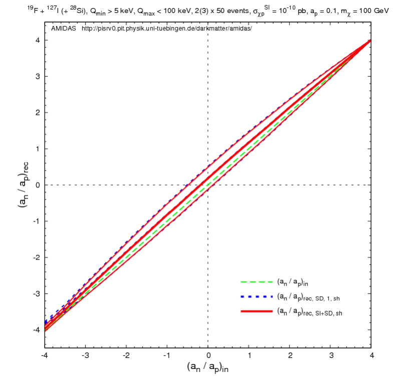
[M. Drees and CLS, arXiv:0903.3300]

Determinations of ratios of WIMP-nucleon cross sections

- Reconstructed $(a_n/a_p)^{SI+SD}_{rec}$ vs. $(a_n/a_p)^{SD}_{rec,1}$

  $({}^{19}F + {}^{127}I + {}^{28}Si$, $Q_{min} > 5$ keV, $Q_{max} < 100$ keV, 3×50 events,
  $\sigma^{SI}_{\chi p} = 10^{-8}/10^{-10}$ pb, $a_p = 0.1$, $m_\chi = 100$ GeV)

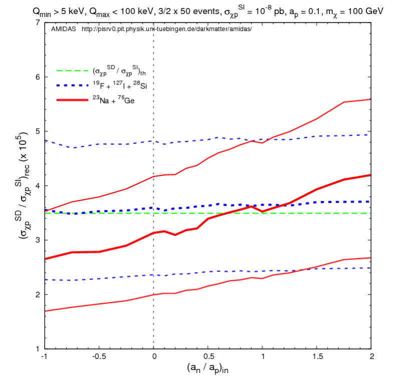


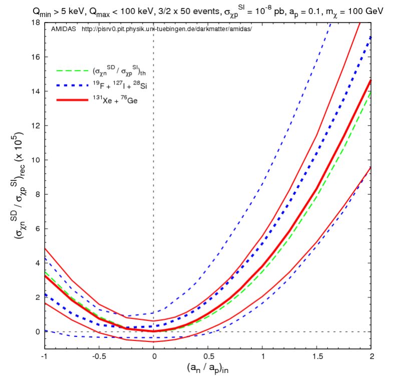


[CLS, JCAP 1107, 005 (2011)]

Determinations of ratios of WIMP-nucleon cross sections

- Reconstructed $\left(\sigma_{\chi p}^{SD}/\sigma_{\chi p}^{SI}\right)_{rec}$ and $\left(\sigma_{\chi n}^{SD}/\sigma_{\chi p}^{SI}\right)_{rec}$
  ($^{19}F + {}^{127}I + {}^{28}Si$ vs. $^{23}Na/{}^{131}Xe + {}^{76}Ge$, $Q_{min} > 5$ keV, $Q_{max} < 100$ keV, $\sigma_{\chi p}^{SI} = 10^{-8}$ pb, $a_p = 0.1$, $m_\chi = 100$ GeV, $3/2 \times 50$ events)





[CLS, JCAP 1107, 005 (2011)]

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- Once two or more experiments with different target nuclei observe positive WIMP signals, we could reconstruct
  - WIMP mass m_χ
  - 1-D velocity distribution $f_1(v)$
  - SI WIMP-proton coupling $|f_{\rm p}|^2$ (with an assumed ρ_0)
  - ratio between the SD WIMP-nucleon couplings $a_{\rm n}/a_{\rm p}$
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- For a WIMP mass of $\mathcal{O}(100\ \mathrm{GeV})$, with only $\mathcal{O}(50)$ events from one experiment and less than $\sim 20\%$ unrejected backgrounds, these quantities could be estimated with statistical uncertainties of $10\% - 40\%$.

Thank you very much for your attention!